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MODELING OF NONSTATIONARY FLOW IN THE RAMJET CHANNEL WITH A DISTRIBUTED PULSE-PERIODIC ENERGY SUPPLY

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A study has been made of the influence of a pulse-periodic supply of energy that is equal to the energy released in combustion of hydrogen in air on the structure of supersonic flow in a channel of variable cross section, which models the ramjet duct. The flow has been modeled based on two-dimensional nonstationary gas-dynamics equations. Different regimes of flow depending on the configuration of the zones of energy supply and the excess-air coefficient have been obtained.

Introduction. Combustion in a supersonic flow has been actively investigated since the 1960s. Numerous experimental and theoretical works on this problem have been published. The phenomenon is used in technology, in particular, in developing economical hypersonic ramjet engines (ramjets). However, the nature of this phenomenon and its influence on flow remain to be elucidated. Experimental investigations, for example, [1] (on an IT-302M pulsed tunnel with Mach numbers M = 6 and 8 at the Institute of Theoretical and Applied Mechanics of the Siberian Branch of the Russian Academy of Sciences), have shown that a system of compression shocks is formed in the model hypersonic-ramjet duct in combustion of hydrogen; this system goes forward and stops in front of the air-intake throat. Evaluations in a one-dimensional approximation which have been performed based on the measurement of the total and static pressures suggest that there can be regimes where the combustion of hydrogen occurs at both subsonic and supersonic velocities. Analogous results have been obtained at this Institute in testing the hypersonic-ramjet model on an AT-303 wind tunnel with Mach number M = 8 [2].

In the present work, we study, based on numerical modeling of plane flow, the gasdynamic aspects of a supply of energy that is equal to the energy released in complete combustion of hydrogen in air. The energy is supplied to a supersonic flow in a channel that models the element of a ramjet and consists of portions of constant and divergent cross sections. Unlike the classical scheme in which the suply of energy to a combustion chamber is continuous due to the burning of the fuel, energy is supplied in a pulse periodic regime, which may model the supply of, for example, radiant energy. Replacement of the chemical reactions by a heat source substantially simplifies the equations. The change in the composition of the mixture and the dependence of the heat capacity and other properties of the medium on temperature and pressure are disregarded. In many cases, such a simplified formulation enables one to reveal the most substantial effects [3]. In ramjet combustion chambers, the mass fraction of the fuel supplied is small as compared to the mass fraction of the air (it has the order of 1.5–3%) [4]. Allowance is made only for the heat source in the energy equation. A two-dimensional problem is solved. It has been considered in [5–7] in the "channel" approximation. Certain results of two-dimensional calculations have been published in [8].

Formulation of the Problem. We model nonstationary flow in a plane channel of variable cross section with a distributed energy supply. The Euler equations are solved in a conservative form for the gas with a constant adiabatic exponent:

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} = \mathbf{Q} \,,$$

$$\mathbf{U} = (\rho, \rho u, \rho v, e) \,, \quad \mathbf{F} = (\rho u, p + \rho u^2, \rho uv, u\,(p + e)) \,, \quad \mathbf{G} = (\rho v, \rho uv, p + \rho v^2, v\,(p + e)) \,,$$

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$$\mathbf{Q} = (0, 0, 0, q)\ .$$

For the gas model in question we have

$$p = (\gamma - 1)\,(e - 0.5\rho\,(u^2 + v^2))\ ,\quad a^2 = T = \frac{\gamma p}{\rho}\ .$$

In pulse periodic energy supply, the quantity q is determined by the expression

$$q = \Delta e\,(x, y)\,g\,(t)\ ,\quad g\,(t) = \sum_m \delta\,(t - m\Delta t)\ .$$

The equations given above are solved in a channel of variable cross section. The shape and dimensions of the channel are indicated in the figures given below. A short initial portion of the channel ($0 \le x < 1$) has a constant cross section with a halfwidth equal to unity. Then the channel smoothly diverges and gives way to a chamber ($6 < x < 16$) in which the supply of energy is carried out ("combustion chamber"; its half-width is $h_2 = 2$). After the chamber, the channel diverges and has an outlet halfwidth equal to three. The total channel length is $l = 26$.

Energy supply is volumetric in character, which corresponds to a good mixing of the fuel and the oxidizer and to homogeneous combustion. The zones of energy supply either have a nearly rectangular shape or are corner-shaped. The latter consist of rectangular portions oriented along the coordinate axes. Energy supply is so rapid each time that we may disregard the change in the density of the gas and in its velocity over the corresponding very short period of time. The density of the gas energy e increases by

$$\Delta e\,(x, y) = \begin{cases} \Delta e\ , & x_{i-1} < x < x_i\ ,\quad y_{i-1} < y < y_i\ ,\quad i = 1, 2, \ldots\ , \\ 0\ , & \text{in the remaining region}\ . \end{cases}$$

The value of Δe is determined by comparison of the power of the supplied energy and the power in its continuous supply in a quantity corresponding to the complete combustion of hydrogen. The energy power supplied (power input) to a unit volume is taken to be

$$q = \rho u \frac{Q}{\Delta x}\ ,\quad Q = \frac{k\mathrm{Hu}}{La_0^2}\ ,$$

where $\Delta x = x_i - x_{i-1}$. This power is supplied uniformly along x in a prescribed range $[x_{i-1}, x_i]$.

In the pulse periodic regime, in accordance with the condition of equality of the energies, the energy

$$\Delta e = \rho u \frac{Q}{\Delta x} \Delta t$$

is supplied. To solve these equations we prescribe the parameters of unperturbed flow at the channel inlet (for $x = 0$). At the outlet ($x = l$), extrapolation is used at supersonic velocities. The nonflow condition — normal velocity component $v_n = 0$ — is set on the upper channel wall (for $y = y(x)$). The symmetry conditions are specified in the plane of symmetry (for $y = 0$): the problem is solved for the upper half of the channel. The parameters of stationary gas flow in the absence of energy supply are used as the initial conditions.

A finite-volume scheme diminishing the total variation (TVD reconstruction) is used to find the numerical solution in intervals between the instants of energy supply. The fluxes at the boundaries of the cells are computed by the method of [9]. Time integration is carried out by the Runge–Kutta method of third order. The computational grid in the physical domain is geometrically adaptive to the channel contour; the grid is rectangular in the canonical domain; the number of computational nodes is 200×140. The stationary solution without energy supply, which is used as the initial condition, is found by the same method, with a relative error of 10^{-6} in the mass flow rate.

Calculation Results. The basic results have been obtained for the following variant. A uniform flow with Mach number M = 2, a gas density of $\rho = \gamma = 1.33$, and a pressure of $p = 1$ is prescribed at the channel inlet. The

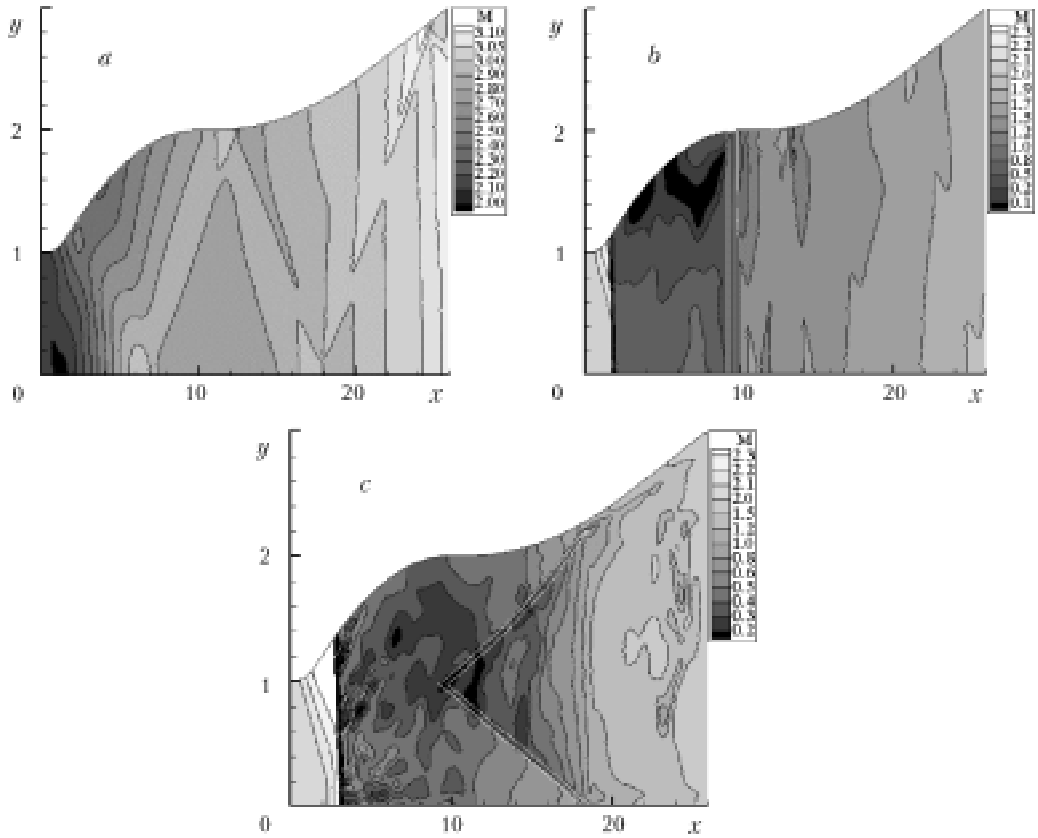


Fig. 1. Mach-number distribution in the channel is stationary flow without energy supply (a), in nearly periodic flow with energy supply in the rectangular zone (b), and in flow with energy supply in the corner-shaped zone (c).

quantity Q determining the power input is taken to be Q = 9.85. These values of the parameters correspond to the flight conditions with Mach number M_∞ = 5.65 at an altitude where the air temperature is T_∞ = 218 K, with a tenfold compression of the cross section of the jet in the air intake. The calculation results given below have been obtained for a value of the period of energy supply of Δt = 0.1.

Figure 1 gives, for the sake of comparison, the distributions of the Mach number in stationary flow without energy supply (Fig. 1a), in nearly periodic flow (Fig. 1b) developing in energy supply in a single rectangular zone perpendicular to the plane of symmetry of the channel and covering its entire cross section, and in energy supply in the zone in the form of a thin corner (Fig. 1c) that nearly covers the channel cross section (the boundaries of the zones of energy supply are shown in white). If we compare this situation to the combustion of hydrogen, the latter solutions have been obtained with an excess-air coefficient equal (Fig. 1b) or close to (Fig. 1c) unity. For better visualization of the details of flow, the scale of y in this figure and in subsequent figures has been enlarged. In stationary flow, the flow is accelerated and the Mach number at the channel inlet exceeds the value M = 3. Weak compression and rarefaction waves are seen in Fig. 1a. Disintegration of an arbitrary discontinuity begins immediately after the energy supply. Short waves propagating upstream and downstream are formed. The gas escapes from the zone of energy supply. In periodic energy supply, the shock wave propagating upstream emerges from the "combustion chamber" and "stops" in the contraction of the channel. This is a direct compression shock; the velocity of the gas behind it is subsonic. In the zone of energy supply, the flow is accelerated to the velocity of sound. In Fig. 1b, it is seen that the M = 1 isoline is adjacent to the right-hand boundary of the zone of energy supply. This value of the Mach number is attained at the upper wall later because of the deceleration of the subsonic flow as a result of the divergence of the channel. Thus, we have energy supply in the subsonic region of flow. The shock wave propagating downstream has emerged from the channel. The flow presented in Fig. 1b has been called nearly periodic, since the position of the direct shock and the parameter distribution in the channel are virtually identical when they are compared at different in-

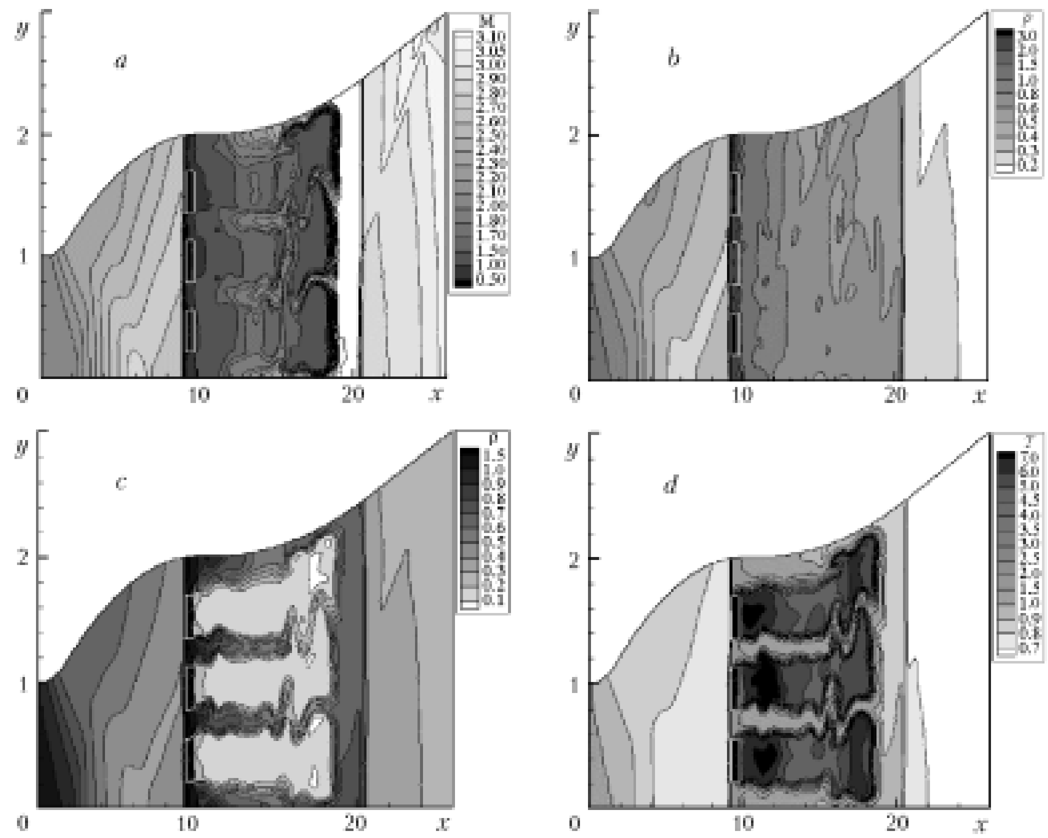


Fig. 2. Distribution of the parameters of flow in energy supply in three zones after 30 periods: a) Mach-number distribution; b) pressure field; 3) density distribution; d) temperature field.

stants of time immediately before the energy supply; however, the rate of flow of the gas through the outlet cross section of the channel experiences slight variations (within a few percent). In energy supply in the zone having the shape of a thin corner (Fig. 1c), the compression shock propagating upstream is also established in the narrower part of the channel; it is weaker but direct, as previously. Flow in the zone of energy supply is subsonic. The entire flow is less regular. These results have been derived after 10,000 periods of energy supply.

In energy supply, we also have disintegration of an arbitrary discontinuity in several rectangular zones not covering the channel cross section. The shock waves from each zone are curvilinear and propagate in all directions. The gas escapes from the zones of energy supply. The shock waves from different energy-supply zones, propagating upstream, merge together, forming a direct compression shock (if the excess-air coefficient is not very large). In periodic energy supply, this compression shock, just as in the cases considered above, emerges from the "combustion chamber" and "stops" in the contraction of the channel. The velocity of the gas behind the direct shock is subsonic. The shock waves propagating downstream also merge into a single shock wave which is removed from the channel with time. In energy supply in three rectangular zones (the excess-air coefficient is equal to approximately two), the distribution of the flow parameters after 30 periods is presented in Fig. 2. The direct compression shock propagating upstream has already been formed, but the distance between it and the zones of energy supply is not appreciable in any way. Flow in front of it is not perturbed by the energy supplied. As a result of energy supply, the gas flow is accelerated to a velocity exceeding more than thrice the velocity of sound (Fig. 2a) and is decelerated in a weak direct compression shock propagating downstream. A contact discontinuity the pressure in whose region is nearly constant propagates behind it (Fig. 2b). The gas flow between this contact discontinuity and the zones of energy supply is broken by contact discontinuities going from the upper and lower boundaries of the zones of energy supply and oriented predominantly along the channel. They are the most pronounced in Fig. 2c and d, where the density and temperature distribution of the gas is shown respectively. These discontinuities are unstable; their oscillation is observed. Trans-

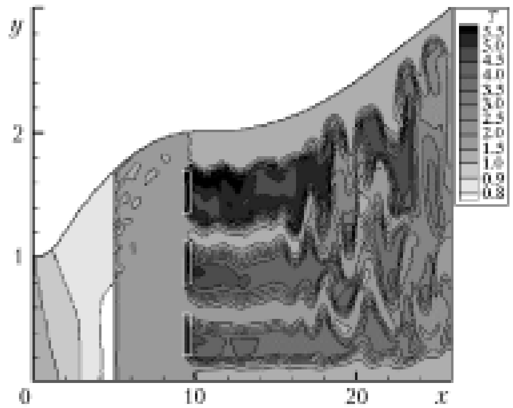
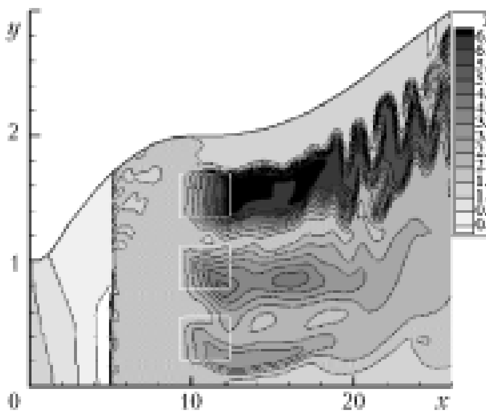


Fig. 3. Temperature field in energy supply in three rectangular zones after 10,000 periods.



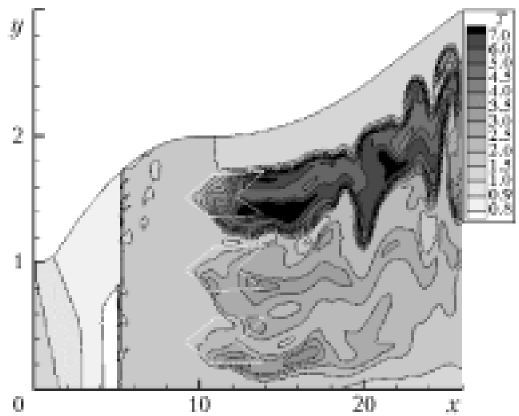


Fig. 4. Temperature field in energy supply in three rectangular zones extended along x after 10,000 periods.

Fig. 5. Temperature field in "periodic" flow with energy supply in three zones having the shape of a thick corner.

verse waves propagate along the contact discontinuities. This instability leads to the impossibility of the periodic regime of flow reaching the steady state, despite the periodicity of energy supply. Figure 3 shows the temperature field after 10,000 periods for this variant. The position of the direct compression shock propagating upstream has been established. The flow pattern in a certain region behind the shock is also virtually constant. However, flow at the channel outlet is very combined in character because of the subsequent development of the oscillatory motion of contact discontinuities. The distance between two successive convexities (or concavities) of the contact surface is two to three times as large as the distance over which a perturbation propagates over a period.

Burning of the fuel occurs at a finite rate, on which the length of the zone of heat release depends. To elucidate the influence of the length of the energy-supply zones on the character of flow we have carried out calculation for energy supply in the zones whose length along x is six times larger than that in the variant in Figs. 2 and 3. Figure 4 gives the corresponding temperature distribution after 10,000 periods, just as Fig. 3. The position of the direct compression shock has virtually not changed. However, the temperature level behind the upper zone of supply has become higher, whereas that behind the lower zone has become lower, which is due to the redistribution of the energy supplied among the zones. Flow in the lower part of the channel has become more regular. We observe, as previously, a considerable instability of the contact discontinuities going from the upper zone of energy supply. A change in the shape of the zones of energy supply, with their area and position being preserved, has, apparently, a small effect on the character of flow. We can seen this by comparing Figs. 4 and 5. In the latter figure, the zones have the shape of

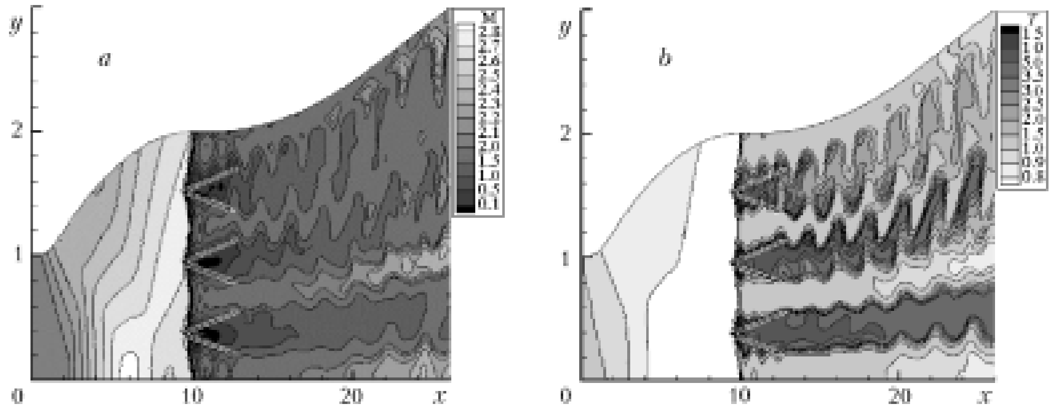


Fig. 6. Distribution of the Mach number (a) and the temperature (b) after 10,000 periods in energy supply in three zones having the shape of a thin corner.

a thick corner. The flow regime becomes "periodic" as far as the rate of flow through the outlet cross section of the channel is concerned.

The character of flow with energy supply differs sharply in three zones having the shape of a thin corner (the excess-air coefficient is also equal to approximately two). Figure 6 gives the Mach-number distribution (Fig. 6a) and the temperature field (Fig. 6b) for such a shape of these zones. The shock wave being established, which propagates upstream, does not separate from the zones of energy supply; it is obviously skew-type. However, the energy is supplied to a subsonic flow. This is seen in Fig. 6a from the distribution of the Mach number immediately before the next energy supply.

Conclusions. It has been established that, in volumetric energy supply that is equal to the energy released in complete combustion of hydrogen in air, we have a significant reorganization of supersonic flow in a channel modeling a ramjet. There are two regimes of flow. The characteristic features of the flow structure in the first regime are a direct compression shock separated from the zone of energy supply upstream, a subsonic region behind it in which "burning" occurs, and a portion of acceleration of the flow and transition to supersonic velocities. Instability of the contact discontinuities separating the gas flows to which the energy is supplied and to which it is not supplied is observed. With energy supply in zones having the shape of a thin corner and an excess-air coefficient of the order of two, we observe another regime whose characteristic feature is a curvilinear "attached" compression shock. Flow behind it is also subsonic, and energy is supplied at subsonic flow velocities.

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NOTATION

a, dimensionless velocity of sound referred to a_0; a_0, velocity of sound in the incident flow; d, halfwidth of the channel at inlet; e, total energy of a unit volume of the gas, referred to $\rho_0 a_0^2$; $\mathbf{F}$, vector function of the flow along the x axis; $\mathbf{G}$, vector function of the flow along the y axis; Hu, calorific value of hydrogen; h_2, halfwidth of the "combustion chamber"; i, No. of the energy-supply zone (or of a portion of the zone in the shape of a corner); k, prescribed coefficient of increase in the energy supplied in a pulsed periodic regime; L, stoichiometric coefficient; l, total channel length referred to d; M, Mach number; M_∞, flight Mach number; m, number of periods of energy supply; p, pressure referred to $\rho_0 a_0^2$; p_0, dimensional pressure at the channel inlet; $\mathbf{Q}$, vector source function; Q, quantity determining the power input; q, power supplied to a unit volume of the gas, referred to $\rho_0 a_0^3/d$; T, dimensionless temperature; T_∞, temperature of air at the flight altitude, K; t, time referred to d/a_0; $\mathbf{U}$, vector function of the quantities sought; u and v, components (referred to a_0) of the gas-velocity vector along the x and y axes respectively; x and y, Cartesian coordinates directed along the channel and across it respectively and referred to d; γ, adiabatic exponent; Δe, energy supplied to a unit volume of the gas; Δt, period of energy supply; Δx, length of the ith zone of energy supply; $\delta(t)$, pulsed Dirac function; ρ, dimensionless density of the gas in the flow, referred to ρ_0; ρ_0, determined from the condition $p_0 = \rho_0 a_0^2$. Subscripts: 0, dimensional parameters at the channel inlet; ∞, parameters at the flight altitude; n, normal.

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